MATH 512 HOMEWORK 3

Due Friday, March 22

Problem 1. *Suppose that $\mathbb{P} * \dot{\mathbb{Q}}$ is a two step iteration.*

(1) *If G is $\mathbb{P}$-generic over V and H is $\dot{\mathbb{Q}}_G$ - generic over $V[G]$, show that $G * H := \{(p, \dot{q}) \mid p \in G, \dot{q}_G \in H\}$ is $\mathbb{P} * \dot{\mathbb{Q}}$-generic over V.*

(2) *Suppose K is $\mathbb{P} * \dot{\mathbb{Q}}$-generic over V and define $G := \{p \in \mathbb{P} \mid (\exists \dot{q})((p, \dot{q}) \in K)\}$ and $H := \{\dot{q}_G \mid (\exists p)((p, \dot{q}) \in K)\}$. Show that G is $\mathbb{P}$-generic over V and H is $\dot{\mathbb{Q}}_G$ - generic over $V[G]$*

Problem 2. *Show that $\mathbb{P} * \dot{\mathbb{Q}}$ has the κ-chain condition if and only if $\mathbb{P}$ has the κ-chain condition and $1_{\mathbb{P}} \Vdash \dot{\mathbb{Q}}$ has the κ-chain condition.*

Problem 3. *Prove the following lemma: Suppose that κ is a regular uncountable cardinal and $\mathbb{P}_\kappa$ is an iteration of length κ, such that at κ we take a direct limit, and $\{\alpha < \kappa \mid \mathbb{P}_\alpha$ is a direct limit $\}$ is stationary. Then if each $\mathbb{P}_\alpha$ has the κ-c.c., $\mathbb{P}_\kappa$ also has the κ-c.c.*

Recall that if U is a normal measure on κ, the Prikry poset to change the cofinality of κ to ω, defined from U has conditions of the form $\langle s, A \rangle$, where s is a finite increasing sequence of ordinals in κ and $A \in U$, and $\langle s', A' \rangle \leq \langle s, A \rangle$ if s is an initial segment of s', $s' \setminus s \subset A$, and $A' \subset A$.

Problem 4. *Suppose that $j : V \to M$ is an elementary embedding with critical point κ. Let U be a normal measure on κ, and let $\mathbb{P}$ be the Prikry poset defined from U. If G is $\mathbb{P}$-generic, show that j cannot be lifted to an embedding $j' : V[G] \to M^*$.*

Problem 5. *(Generalized Prikry lemma) Let $\mathbb{P}$ be the Prikry poset defined from a normal measure U on κ, and suppose that D is a dense open subset of $\mathbb{P}$. Show that for all $\langle s, A \rangle$, there is some n and a measure one set $A' \subset A$, such that every $\langle t, B \rangle \leq \langle s, A' \rangle$ with $|t| = |s| + n$ is in D.*